Many “faces” of Mirror Symmetries

Huajia Wang
University of Illinois at Urbana Champaign

S. Kachru, M. Mulligan, G.Torroba, H. Wang, arXiv: 1608.05077

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Introduction/Motivation

Duality: **equivalence between descriptions non-perturbatively related**

Examples:

2+1 dimensions: Peskin-Dasgupta-Halperin (PDH) duality (1978, 1981)

$$S_{\mathrm{XY}} = \int d^3x |(\partial_\mu - iA_\mu)\phi|^2 - V(\phi) \quad \Rightarrow \quad \boxed{U(1)_J \; : \; J^\mu \propto \epsilon^{\mu\nu\rho} f_{\nu\rho}}$$

$$\Leftarrow \quad S_{\mathrm{A.H.}} = \int d^3x |(\partial_\mu - i\underbrace{a_\mu}_{\text{dynamical gauge field}})\Phi|^2 - \tilde{V}(\Phi) + \frac{1}{2\pi}\epsilon^{\mu\nu\rho} A_\mu \partial_\nu a_\rho$$

Particle-vortex duality: $\phi \quad \Longleftrightarrow$ Monopole operator $\oint f_{\mu\nu} = 2\pi$

Introduction/Motivation

Recently: **particle-hole symmetry + composite fermion + 1/2 Landau level**

conjecture (Son, 14):

$$S_{\text{free}} = \int dx^3 i\bar{\psi}\gamma^\mu \left(\partial_\mu - iA_\mu\right)\psi$$

$$\boxed{U(1)_J \;:\; J^\mu \propto \epsilon^{\mu\nu\rho} f_{\nu\rho}}$$

$$S_{\text{QED}} = \int dx^3 i\bar{\Psi}\gamma^\mu \left(\partial_\mu - i\underset{\substack{\uparrow \\ \text{dynamical gauge field}}}{a_\mu}\right)\Psi + \frac{1}{4\pi}\epsilon^{\mu\nu\rho} A_\mu \partial_\nu a_\rho$$

(fermionic version)

Particle-vortex duality: $\psi \Longleftrightarrow$ Monopole operator (2-fluxes) $\oint f_{\mu\nu} = 4\pi$

Can be understood as arising at the boundary of 3+1 bulk topological order under S duality. (Wang, Senthil; Metlitski and Vishwanath, 15)

Introduction/Motivation

Bosonization/Fermionization dualities: (Aharon, 16)

N_f **fermions +** $U(k)_{-N+\frac{N_f}{2},-N+\frac{N_f}{2}}$		N_f **scalars +** $SU(N)_k$
N_f **fermions +** $SU(k)_{-N+\frac{N_f}{2}}$	$\Longleftrightarrow$	N_f **scalars +** $U(N)_{k,k}$
N_f **fermions +** $U(k)_{-N+\frac{N_f}{2},-N-k+\frac{N_f}{2}}$		N_f **scalars +** $SU(N)_{-N+\frac{N_f}{2}}$

Main Evidences:

large N, k limit: dual to the same higher-spin (Vasiliev) theories of gravity in AdS4

massive deformations: level-rank dualities

Introduction/Motivation

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N_f **fermions +** $U(k)_{-N+\frac{N_f}{2},-N-k+\frac{N_f}{2}}$	$\Longleftrightarrow$	N_f **scalars +** $SU(N)_{-N+\frac{N_f}{2}}$

special case of interest: $N_f = N = k = 1$

$$\bar{\psi} i \not{D}_a \psi - \frac{1}{8\pi} a da - \frac{1}{2\pi} A da \quad \Longleftrightarrow \quad |D_A \phi|^2 - V(\phi) + \frac{1}{4\pi} A dA$$

$$\bar{\Psi} i \not{D}_A \Psi - \frac{1}{8\pi} A dA \quad \Longleftrightarrow \quad |D_{-a} \varphi|^2 - V(\varphi) + \frac{1}{4\pi} a da - \frac{1}{2\pi} A da$$

Introduction/Motivation

"Web of dualities": **(E. Witten, et al; A. Karch, at al 16)**

$$Z_{\text{scalar+flux}}(A) = Z_{\text{fermion}}(A)\, e^{-\frac{i}{8\pi}\int AdA}$$

$$Z_{\text{scalar-QED}}(A) = Z_{\text{scalar}}(A)$$

PDH's duality

$$Z_{\text{QED}}(A) = Z_{\text{fermion}}(A)$$

Son's duality

$$Z_{\text{fermion+flux}}(A) = Z_{\text{scalar}}(A)\, e^{\frac{i}{4\pi}\int AdA}$$

by formal manipulation of path-integrals

"proving one automatically proves other three!"

Introduction/Motivation

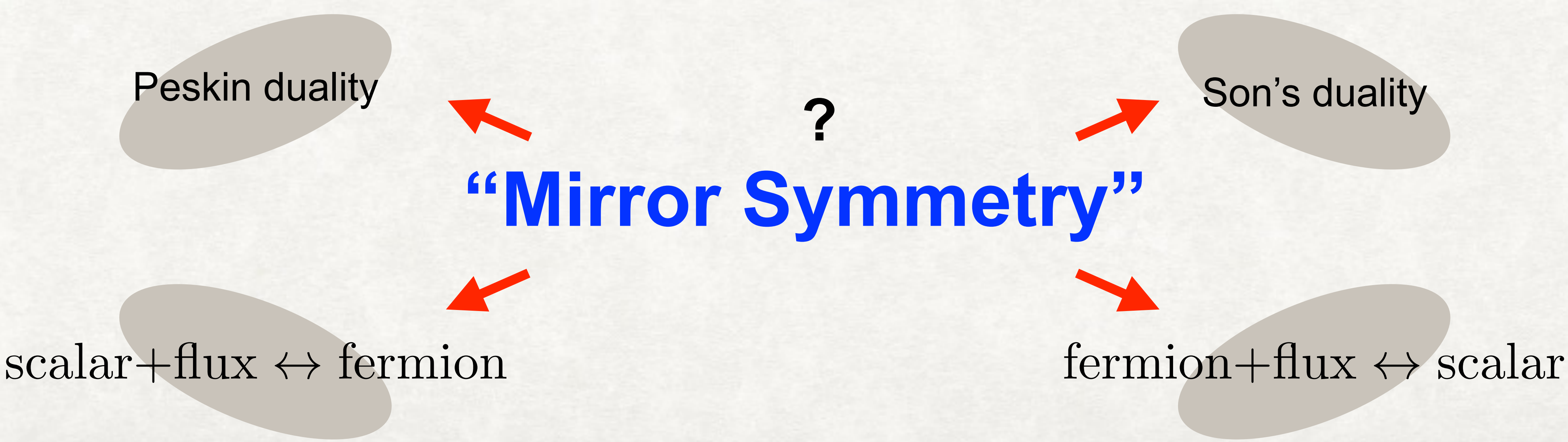
“Web of dualities”: **(E. Witten, et al; A. Karch, at al 16)**

Proving any one of the dualities is difficult, due to lack of control parameters in the strongly coupled theories.

numerical evidence (S. Geraedts, et’al, 15); wire construction (D. Mross, et’al, 17);

lattice construction (J. Chen, et’al, 17)

Goa of the talk: can they be connected with better understood dualities (mirror symmetry)?



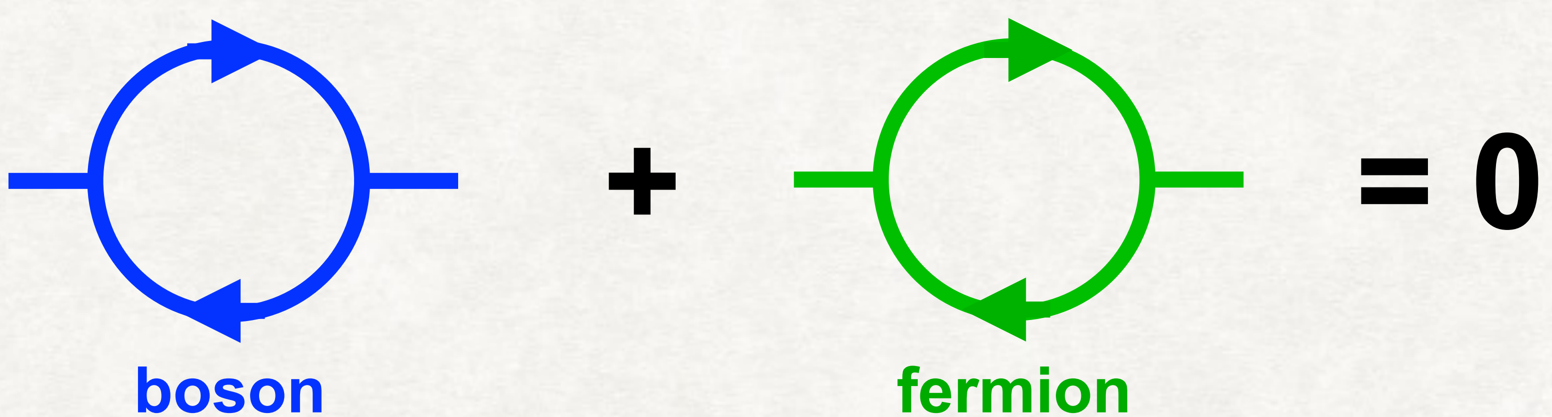
Mirror symmetry: Exact dualities

Key ingredient: SUPERSYMMETRY (SUSY)

make fun of SUSY?

An organizing principle for quantum corrections:

Symmetry: boson ⟷ fermion



SUSY in condensed matter systems:

S.S. Lee'06; T. Grover, at al' 13;

Mirror symmetry:

Chiral mirror symmetry: **(D. Tong, 00; S. Kachru et'al, 16)**

"Theory A" ⟷ **"Theory B"**

$$\mathcal{L}_A \propto |\partial_\mu \phi|^2 + i\bar{\psi}\partial\!\!\!/\psi$$

$$\mathcal{L}_B \propto \frac{1}{g^2}\mathcal{K}_{\text{kin}}(a,\lambda,\sigma,D) + |D_{-a}\varphi|^2 + i\bar{\Psi}D\!\!\!\!/_{-a}\Psi$$

$$+\frac{1}{8\pi}ada +$$ **"SUSY completions"**

Free theory!

"SUSY-multiplets"

hypermultiplet: $\{\phi,\psi\}$, $\{\varphi,\Psi\}$ **SUSY extension of charged field**

vectormultiplet: $\{a_\mu,\lambda,\sigma,D\}$ **SUSY extension of gauge field**

SUSY completions of interactions

e.g. **1/2 CS gauge coupling** $\rightarrow \dfrac{1}{8\pi}\left(ada + 2D\sigma + \bar{\lambda}\lambda\right)$

Mirror symmetry:

Identify global symmetries:

"Theory A" ⟷ **"Theory B"**

	$U(1)_J$	$U(1)_R$
ϕ	1	1
ψ	1	0

	$U(1)_J$	$U(1)_R$	$U(1)_g$
φ	0	0	-1
Ψ	0	-1	-1
$e^{2\pi i\gamma/g^2}$	1	0	0
σ	0	0	0
λ	0	-1	0

Background vector multiplet: $\hat{A}_\mu \to \{\hat{A}_\mu, \hat{\sigma}, \hat{D}\}$

Deforming away from Mirror symmetry:

Break SUSY: **"Mirror Symmetry"**

?

Peskin duality

Son's duality

scalar+flux $\leftrightarrow$ fermion

fermion+flux $\leftrightarrow$ scalar

Deforming away from Mirror symmetry:

Break SUSY: “Mirror Symmetry”

?

Peskin duality

Son’s duality

scalar+flux $\leftrightarrow$ fermion

fermion+flux $\leftrightarrow$ scalar

Turn on backgrounds: $\{\hat{A}^J_\mu, \hat{\sigma}^J, \hat{D}^J\}$ **+** $\{\hat{A}^R_\mu, \hat{\sigma}^R, \hat{D}^R\}$

Deforming away from Mirror symmetry:

Break SUSY: **"Mirror Symmetry"**

?

Peskin duality

Son's duality

scalar+flux $\leftrightarrow$ fermion

fermion+flux $\leftrightarrow$ scalar

Lorentz symmetry

Turn on backgrounds: $\{\cancel{\hat{A}^J_\mu}, \hat{\sigma}^J, \hat{D}^J\}$ **+** $\{\cancel{\hat{A}^R_\mu}, \hat{\sigma}^R, \hat{D}^R\}$

Deforming away from Mirror symmetry:

Break SUSY: “Mirror Symmetry”

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Peskin duality

Son’s duality

scalar+flux $\leftrightarrow$ fermion

fermion+flux $\leftrightarrow$ scalar

Lorentz symmetry

Turn on backgrounds: $\{\cancel{\hat{A}^J_\mu}, \hat{\sigma}^J, \hat{D}^J\}$ **+** $\{\cancel{\hat{A}^R_\mu}, \hat{\sigma}^R, \hat{D}^R\}$

One simple choice: $\{\hat{\sigma}^J, \hat{D}^J\}$

Theory A “minimal coupling”

$$\left(\hat{\sigma}_J^2 - \hat{D}_J\right)|\phi|^2 + \hat{\sigma}_J \bar{\psi}\psi \qquad \propto \bar{\psi}\not{\hat{A}}^J\psi + \ldots$$

$\hat{\sigma}_J$: **SUSY mass**

$\hat{D}_J$: **mass-split**

Theory B “B.F. coupling”

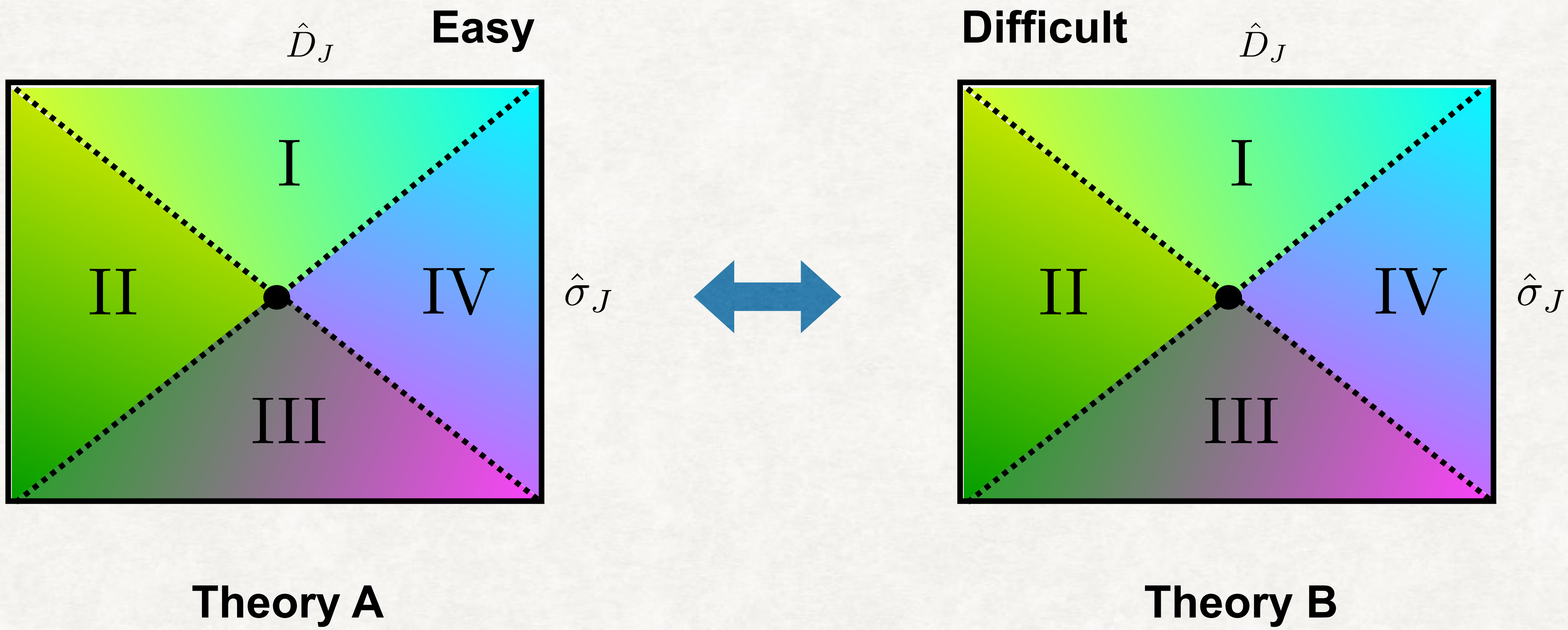
$$\frac{1}{2\pi}\left(\hat{D}_J\sigma + \hat{\sigma}_J D\right) \qquad \propto da\hat{A}_J$$

tad-poles for gauge multiplet

Deforming away from Mirror symmetry:

Strategy:

Step1: identify massive phases (I, II, III, IV)

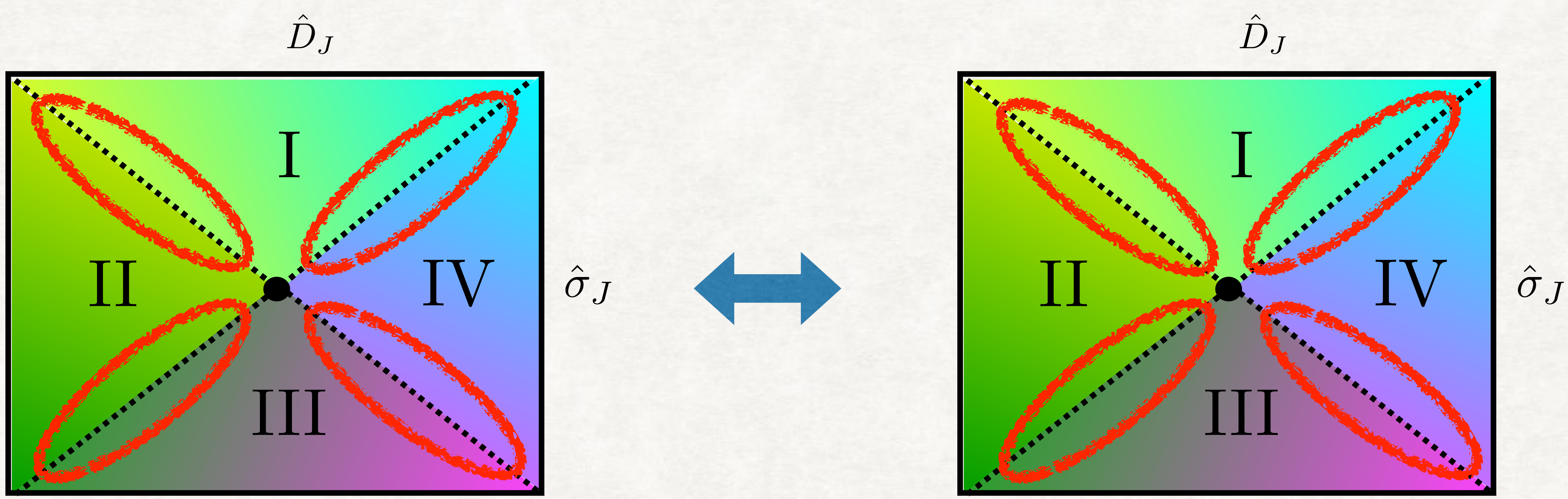


characterized by topological responses: $\mathcal{L}_{A,B} \propto K_{ij}\hat{A}^i d\hat{A}^j,\ i,j \in \{J,R\}$

Deforming away from Mirror symmetry:

Strategy:

Step2: identify critical phases (I/II, II/III, III/IV, IV/I)



Theory A **Theory B**

$$\left\{ \mathcal{L}_A(\mathrm{I/II}) \equiv \mathcal{L}_\mathrm{B}(\mathrm{I/II})\,,\ \mathcal{L}_A(\mathrm{II/III}) \equiv \mathcal{L}_\mathrm{B}(\mathrm{II/III})\ \ldots \right\} \overset{?}{\longleftarrow}$$ **web of dualities**

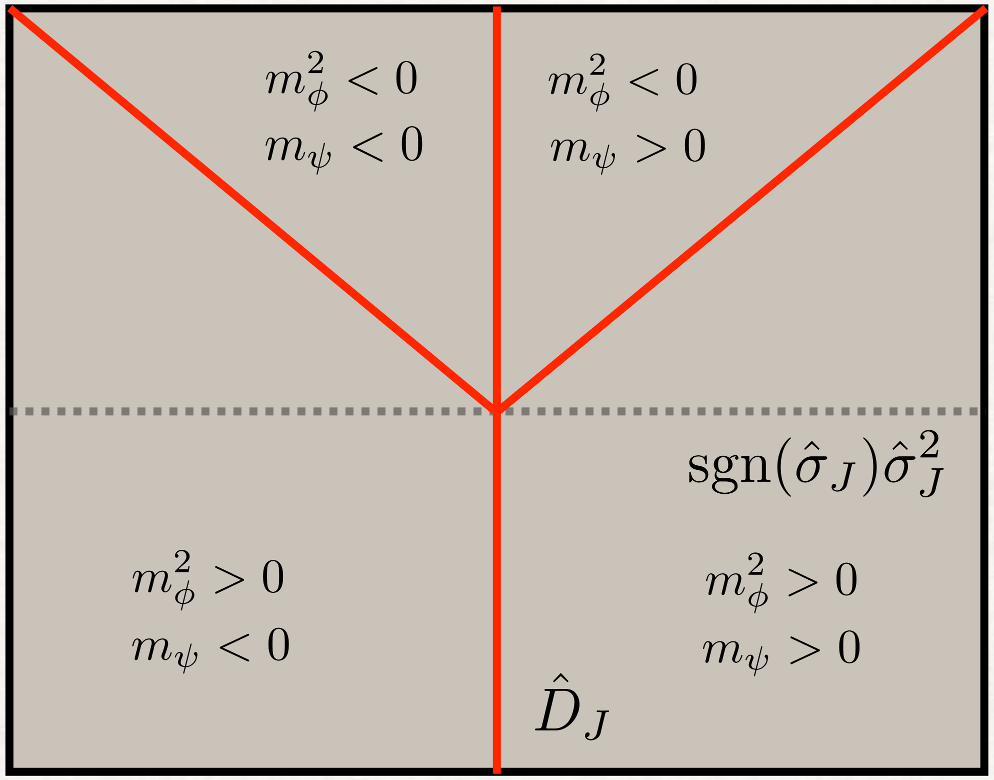
Massive phases

Theory A is free, easy to identify responses

$$\mathcal{L}_A \propto |D_{\hat{A}_J+\hat{A}_R}\phi|^2 + i\bar{\psi}\not{D}_{\hat{A}_J}\psi$$
$$-m_\phi^2|\phi|^2 - m_\psi\bar{\psi}\psi \; -\frac{1}{8\pi}\hat{A}_J d\hat{A}_J$$

$$\boxed{\begin{aligned} m_\phi^2 &= \hat{\sigma}_J^2 - \hat{D}_J \\ m_\psi &= \hat{\sigma}_J \end{aligned}}$$

$\Rightarrow$



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integrating out massive fermion

$$m_\psi > 0 \rightarrow \mathcal{L}_A \propto 0$$
$$m_\psi < 0 \rightarrow \mathcal{L}_A \propto -\frac{1}{4\pi}\hat{A}_J d\hat{A}_J$$

integrating out massive scalar

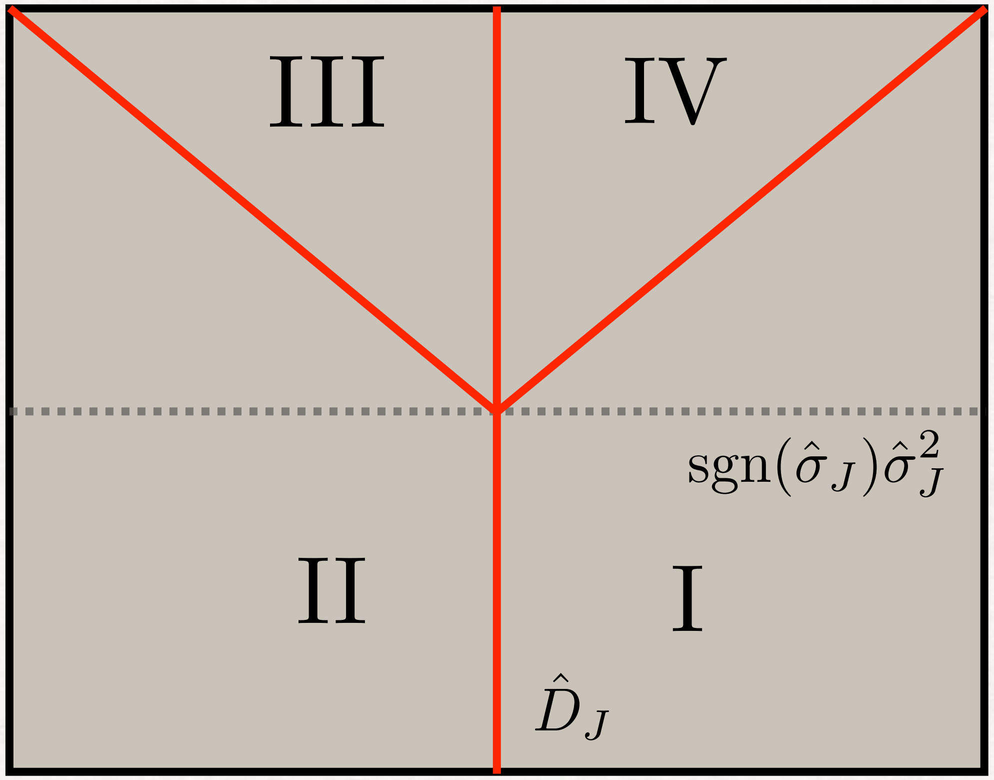
$$m_\phi^2 > 0 \rightarrow \text{ nothing}$$
$$m_\phi^2 < 0 \rightarrow \hat{A}_J + \hat{A}_R = 0$$

Massive phases

Theory A is free, easy to identify responses

$$\mathcal{L}_A \propto |D_{\hat{A}_J+\hat{A}_R}\phi|^2 + i\bar{\psi}\not{D}_{\hat{A}_J}\psi$$
$$-m_\phi^2|\phi|^2 - m_\psi\bar{\psi}\psi \; - \frac{1}{8\pi}\hat{A}_J d\hat{A}_J$$

$$\boxed{\begin{aligned} m_\phi^2 &= \hat{\sigma}_J^2 - \hat{D}_J \\ m_\psi &= \hat{\sigma}_J \end{aligned}}$$



Responses:

$$\mathrm{I}: \quad \mathcal{L}_A = 0$$

$$\mathrm{II}: \quad \mathcal{L}_A = -\frac{1}{4\pi}\hat{A}_J d\hat{A}_J$$

$$\mathrm{III}: \quad \mathcal{L}_A = -\frac{1}{4\pi}\hat{A}_J d\hat{A}_J - \frac{b}{2\pi}\left(\hat{A}_J + \hat{A}_R\right)$$

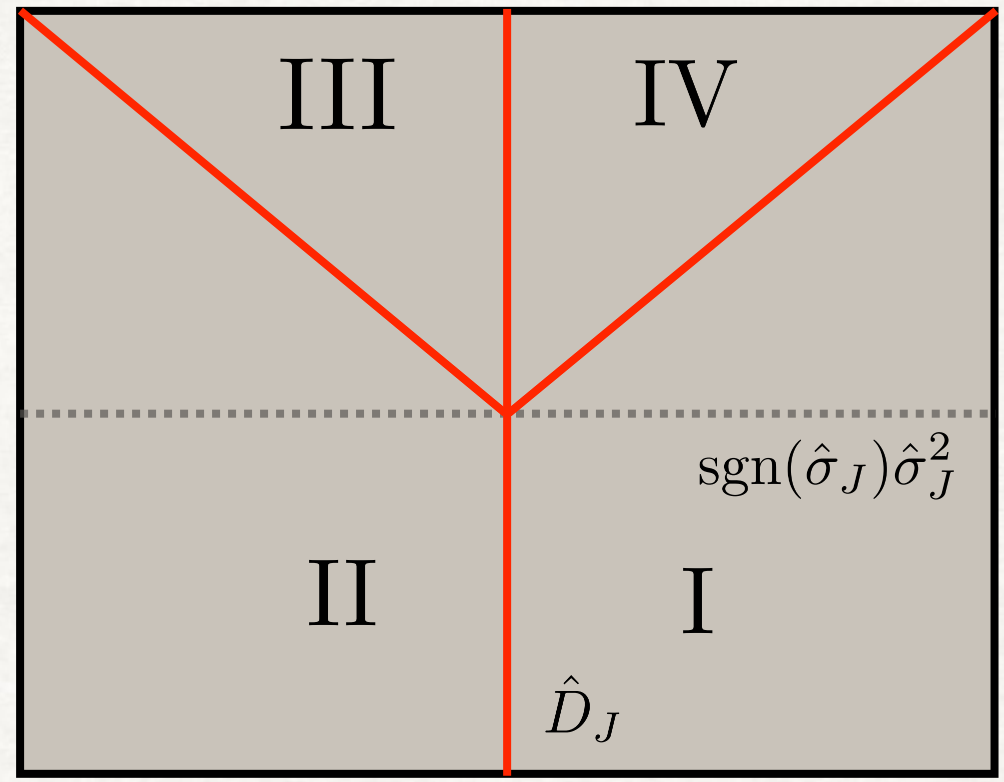
$$\mathrm{IV}: \quad \mathcal{L}_A = -\frac{b}{2\pi}\left(\hat{A}_J + \hat{A}_R\right)$$

Massive phases

Theory B follows from duality: $Z_A\left(\hat{\sigma}_J,\hat{D}_J\right)=Z_B\left(\hat{\sigma}_J,\hat{D}_J\right)$

$$\mathcal{L}_B \propto \frac{1}{g^2}\mathcal{K}_{\text{kin}}\left(a,\lambda,\sigma,D\right)+|D_{-a}\varphi|^2$$
$$+i\bar{\Psi}\not{D}_{-a}\Psi+\frac{1}{8\pi}\left(ada+2D\sigma+\bar{\lambda}\lambda\right)$$
$$-\frac{1}{2\pi}\left(\hat{D}_J\sigma+\hat{\sigma}_J D\right)-\sigma^2|\varphi|^2-\sigma\bar{\Psi}\Psi$$
$$+\lambda\Psi\varphi+h.c.-\frac{1}{4\pi}a\left(2d\hat{A}_J+d\hat{A}_R\right)$$

difficult to analyze in its own!



Responses:

$$\text{I}:\quad \mathcal{L}_B=0$$

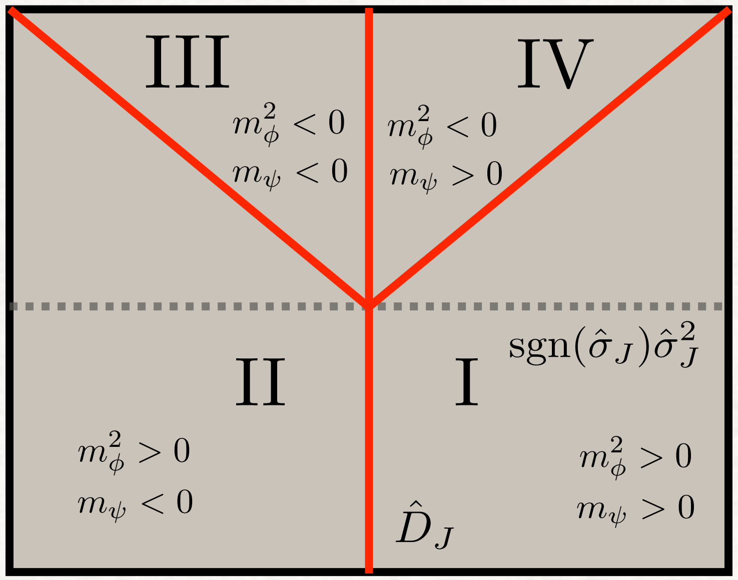
$$\text{II}:\quad \mathcal{L}_B=-\frac{1}{4\pi}\hat{A}_J d\hat{A}_J$$

$$\text{III}:\quad \mathcal{L}_B=-\frac{1}{4\pi}\hat{A}_J d\hat{A}_J-\frac{b}{2\pi}\left(\hat{A}_J+\hat{A}_R\right)$$

$$\text{IV}:\quad \mathcal{L}_B=-\frac{b}{2\pi}\left(\hat{A}_J+\hat{A}_R\right)$$

Critical phases

Theory A



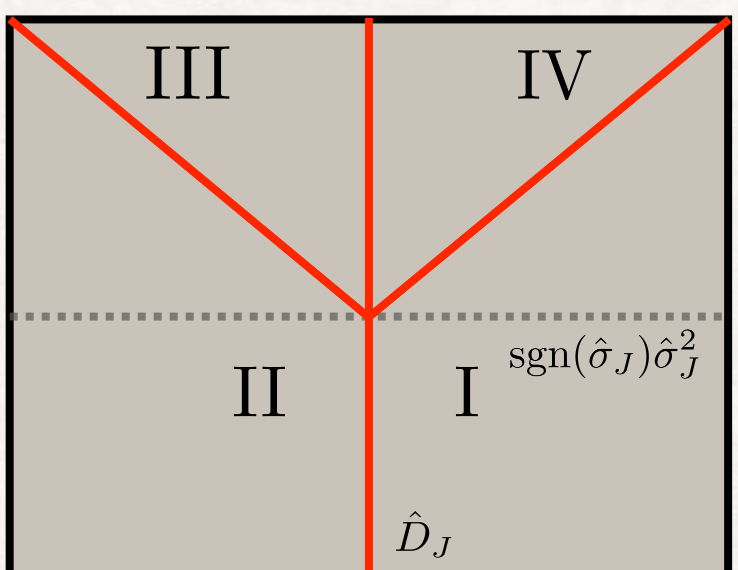
$\mathrm{I}/\mathrm{II}: \quad m_\psi = 0$ **free fermion**

$\mathrm{II}/\mathrm{III}: \quad m_\phi^2 = 0$ **Wilson-Fisher**

$\mathrm{III}/\mathrm{IV}: \quad m_\psi = 0$ **free fermion**

$\mathrm{IV}/\mathrm{I}: \quad m_\phi^2 = 0$ **Wilson-Fisher**

Theory B



$$\left.\begin{array}{r} \mathrm{I}/\mathrm{II}: \\ \mathrm{II}/\mathrm{III}: \\ \mathrm{III}/\mathrm{IV}: \\ \mathrm{IV}/\mathrm{I}: \end{array}\right\} \; ?$$

Clue: consistent with transitions in topological responses

Critical phases

Theory B : **parametrize the IR by effective mass parameters:**

$$\mathcal{L}_B \propto \frac{1}{g^2}\mathcal{K}_{\text{kin}}\left(a,\lambda,\sigma,D\right) + |D_{-a}\varphi|^2$$
$$+ i\bar{\Psi}\not{D}_{-a}\Psi + \frac{1}{8\pi}\left(ada + 2D\sigma + \bar{\lambda}\lambda\right)$$
$$-\frac{1}{2\pi}\left(\hat{D}_J\sigma + \hat{\sigma}_J D\right) - \sigma^2|\varphi|^2 - \sigma\bar{\Psi}\Psi$$
$$+\lambda\Psi\varphi + h.c. - \frac{1}{4\pi}a\left(2d\hat{A}_J + d\hat{A}_R\right)$$

IR

$$\mathcal{L}_B \propto \ldots + m_\varphi^2|\varphi|^2 + \bar{\Psi}_f M_f \Psi_f$$

eigenvalues

$$\Psi_f = (\Psi, \lambda) \qquad \left(m_{f_+}, m_{f_-}\right)$$

Responses (in terms of IR mass parameters):

$$\boxed{m_\varphi^2 > 0 :}$$

$$\mathcal{L}_B \propto \frac{\text{sgn}(m_{f_+})}{8\pi}(a + \hat{A}_R)d(a + \hat{A}_R) + \frac{1}{8\pi}ada$$
$$+\frac{\text{sgn}(m_{f_-})}{8\pi}\hat{A}_R d\hat{A}_R - \frac{1}{4\pi}a\left(2d\hat{A}_J + d\hat{A}_R\right)$$

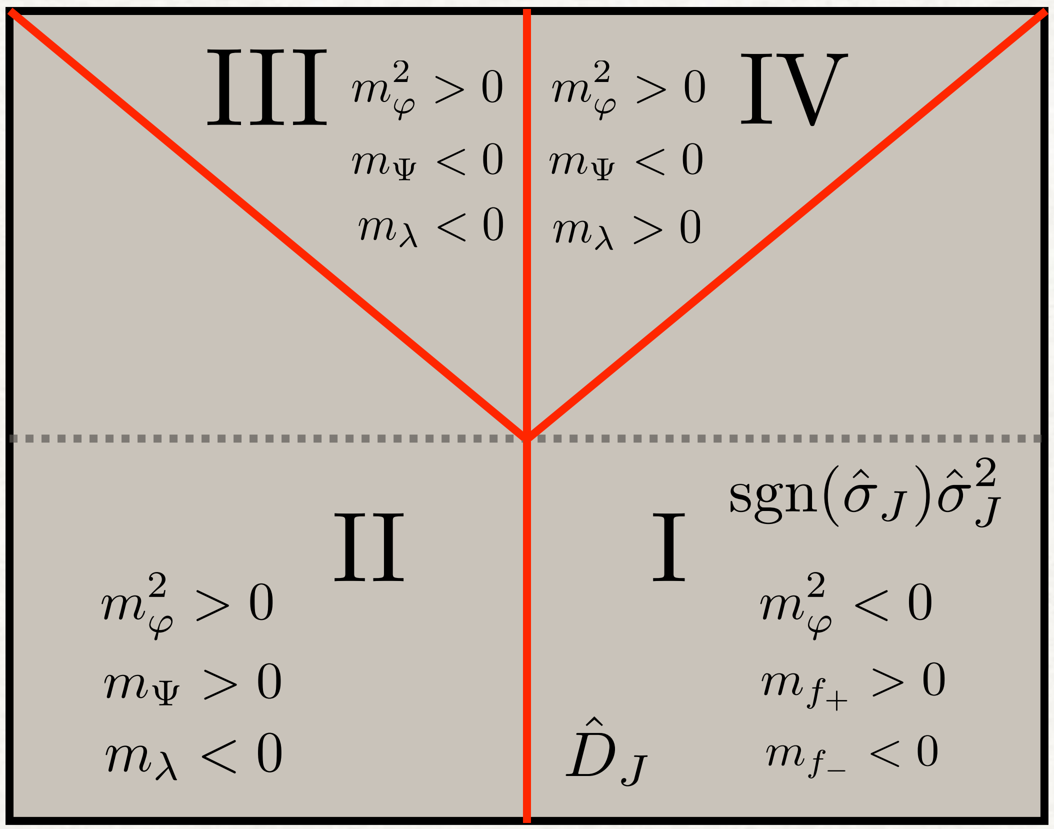
$$\boxed{m_\varphi^2 < 0 : a \to 0}$$

$$\mathcal{L}_B \propto \frac{1}{8\pi}\left[\text{sgn}(m_{f_+}) + \text{sgn}(m_{f_-})\right]\hat{A}_R d\hat{A}_R$$

Critical phases

Theory B :

consistency with responses in massive phases uniquely fix the signs:



$\mathrm{I/II}: \quad m_\varphi^2 = 0$ **sQED**

$\mathrm{II/III}: \quad m_\Psi = 0$ **fQED**

$\mathrm{III/IV}: \quad m_\lambda = 0$ **free fermion**

$\mathrm{IV/I}: \quad m_\varphi^2 = 0$ **sQED**

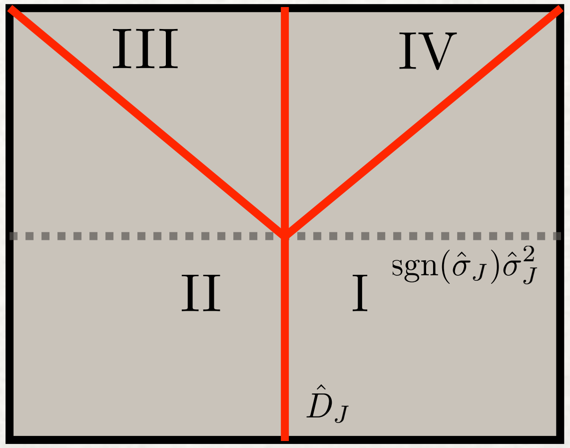
Critical phases

Matching:

	Theory A	Theory B	
I/II :	free fermion	sQED	scalar+flux $\leftrightarrow$ fermion
II/III :	Wilson-Fisher	fQED	fermion+flux $\leftrightarrow$ scalar
III/IV :	free fermion	free fermion	trivial duality
IV/I :	Wilson-Fisher	sQED	Peskin duality

<u>Conclusion:</u>

“web of dualities" can be embedded as deformations of a multi-critical “parent” duality: mirror symmetry



Possible extensions:

More general SUSY-breaking deformations

Other realizations of mirror symmetries (e.g. deconfined criticality?)

Non-relativistic limits

Thank you!